ABSTRACT
This paper investigates performance of a low complexity, noncoherent communication system with power and bandwidth constraints. Towards this end, continuous phase modulation (CPM) using soft-decision differential phase detection and Viterbi decoding is proposed. Careful selection of CPM parameters allows lowering the required $E_b/N_0$, while achieving a targeted spectral efficiency. Simulations at 5 bps/Hz for modulation orders $M = 2, 4, 8$ and 16 in a satellite mobile channel show our proposed system outperforms (in terms of energy efficiency and error rate) some selected popular noncoherent receivers. Also, its error rate is only marginally worse than the optimum coherent receiver, while achieving a considerable reduction in complexity.

INTRODUCTION
Since the available frequency spectrum is limited, it is necessary to investigate modulation schemes and corresponding receiver designs that perform well under bandwidth constraints. CPM [1] is ideally suited for bandwidth constrained applications due to its small spectral side lobes and fast spectral roll-off. Also, the constant envelope property permits the use of power efficient nonlinear amplifiers. The memory inherent in this type of modulation allows maximum likelihood (ML) sequence detection at the receiver [1, 2]. However, the use of ML coherent receivers for wireless military applications has two main limitations, high complexity and sensitivity to phase estimation errors. Low complexity noncoherent detectors are hence preferred in such systems, despite causing some performance loss.

Korn in [3] investigates the limiter discriminator integrator detector (LDID) with and without decision feedback as a low complexity noncoherent receiver for Gaussian minimum shift keying (GMSK) in a satellite mobile (Rician) channel (SMC). In [3] it is shown that decision feedback is effective only for very small bandwidths of the Gaussian filter ($B_g$) and for Rician factor ($K$) $\geq$ 6 dB. In [4], the error probability for a system using GMSK with $\{1, 2\}$-bit differential phase detection (DPD) in a SMC is derived. Similar to [3], DPD with feedback has also been investigated. Instead of suppressing GFSK induced ISI using feedback ([3], [4]), it can be exploited to design ML noncoherent receivers [5]. The limiter discriminator detector and DPD with Viterbi decoding has been investigated in [6] for GMSK in AWGN, where it is shown that noncoherent detectors with Viterbi decoding perform better than those using decision feedback. The soft-decision phase detector with Viterbi decoding was introduced in [7] as a low complexity, coherent receiver. Results for different partial response CPM in AWGN show that this detector performs within 2 dB of the optimum coherent receiver. When the phase is unknown (or cannot be easily estimated) at the receiver, [8] proposes the noncoherent soft-decision DPD with Viterbi decoding (SD-DPD/VD) for convolutional coded BFSK and DPSK.

This paper investigates low complexity, noncoherent receiver design under high spectral efficiencies for a SMC. Our work differs from some well known existing papers on noncoherent receivers ([3], [4], [5], [6], [8], [9], [10], [11]) for CPM as follows. Instead of emphasizing only the noncoherent receiver, we seek to make improvements at the transmitter side as well. At the transmitter, Gaussian Frequency Shift Keying (GFSK) is used due to superior spectral characteristics. An exhaustive search is performed to find GFSK parameters (modulation index $h$, and filter bandwidth $B_g$) for $M = 2, 4, 8$ and 16 that meet a required spectral efficiency and give the best possible tradeoff between error rate and transmitter power. At the receiver, we use SD-DPD/VD and show that for the selected...
$M$-GFSK parameters, it outperforms LDID, LDID with Viterbi decoding (LDID/VD) and (single bit) DPD with Viterbi decoding (DPD/VD). The performance of SD-DPD/VD is also compared with the ML coherent receiver. The coherent receiver is however, sensitive to the carrier phase estimation and it is shown that when the estimation error is large, the SD-DPD/VD performs better, at higher $E_b/N_c$. The tradeoffs between performance and complexity between the different receivers is also discussed. Finally, similar to [12], we show that using rate $1/2$ convolutional codes at high spectral efficiencies causes performance loss.

**SYSTEM MODEL**

The system model is shown in Fig. 1. Let $\mathbf{u} \in \{0,1\}^O$ be the vector of message bits. Without channel coding, $\log_2(M)$ bits of $\mathbf{u}$ are mapped to one of $M$ symbols (natural mapping). This sequence of symbols is denoted by $\mathbf{a} \in \{\pm 1, \pm 3, ..., \pm (M - 1)\}^L$, where, $L = \lceil O/\log_2 M \rceil$. With channel coding, $\mathbf{u}$ is passed through a binary encoder to produce a codeword $\mathbf{b'} \in \{0,1\}^N$ which is interleaved by a permutation matrix $\Pi$ to produce the bit-interleaved codeword $\mathbf{b} = \mathbf{b'}\Pi$. $\log_2(M)$ bits of $\mathbf{b}$ are now mapped to one of $M$ symbols (natural mapping) to give $\mathbf{a} \in \{\pm 1, \pm 3, ..., \pm (M - 1)\}^L$, where now, $L = \lceil N/\log_2 M \rceil$. The $M$-ary, baseband GFSK signal in the interval $nT \leq t \leq (n+1)T$ is [2]

$$x(t, \mathbf{a}) = \sqrt{2E_s/T} \exp \left[ -j \varphi(t, \mathbf{a}) \right]$$

(1)

with symbol energy $E_s$ and symbol period $T$. Assuming the GFSK ISI extends to $Z$ symbols, the phase of the GFSK signal can be written as

$$\varphi(t, \mathbf{a}) = \theta_n + \theta(t; \mathbf{a}) + \theta(t; a_n)$$

(2)

where

$$\theta_n = \pi h \sum_{i=-\infty}^{n-Z-1} a_i$$

$$\theta(t; \mathbf{a}) = 2\pi h \sum_{i=n-Z}^{n-1} a_i \int_{-\infty}^{t} g(\tau - iT)d\tau$$

and

$$\theta(t; a_n) = 2\pi h a_n \int_{-\infty}^{t} g(\tau - nT)d\tau$$

$g(t)$ is the response of the Gaussian shaping filter to a rectangular pulse of duration $T$,

$$g(t) = \left[ Q(cB_2(t - 5T/2)) - Q(cB_2(t - 3T/2)) \right] / 2T$$

where, $c = 7.546$ and $B_2T$ is the normalized 3 dB bandwidth of the filter. From the power spectrum of the signal, the 99% ($B_{99}$) signal bandwidth (normalized by $T$) is found. An exhaustive search is performed at different $M$ for values of $h$ and $B_2T$ that meet a given spectral efficiency. The modulated signal $\mathbf{x}$ at these values is then transmitted through a SMC channel. The signal at the output of the Rician channel is

$$r(t, \mathbf{a}) = \left( \sqrt{2P_s} + \sqrt{P_d} \xi(t) \right) \exp(j \varphi(t, \mathbf{a})) + n(t)$$

(3)

$P_s$ is the power of the direct signal component, $P_d$ is the power of the diffused component, with $K = P_s/P_d$. $\xi(t)$ is a zero mean, complex Gaussian fading coefficient and $n(t)$ is additive, zero mean Gaussian noise. The assumptions are that the channel is memoryless and $\xi$ remains constant during each symbol interval. $r(t, \mathbf{a})$ is passed through a front end receiver filter (not shown in Fig. 1) that removes the out of band noise. If the normalized filter bandwidth ($BT$) is greater than $B_{99}$, the signal remains sufficiently undistorted by the filter. From [3] and [6], the phase of the filtered received signal is

$$\phi(t, \mathbf{a}) = \varphi(t, \mathbf{a}) + \eta(t)$$

(4)

where

$$\eta(t) = \arctan \frac{n_c(t)}{n_a(t) + \sqrt{2}\rho}$$

and, $n_c(t)$ and $n_a(t)$ are the in-phase and quadrature phase components of $n(t)$. Also

$$\rho = \frac{P_s}{P_d + P_n}$$

where, $P_n$ is the power of the additive noise component. The filtered signal is passed to the SD-DPD/VD, which gives the ML estimate of the transmitted symbols $\hat{\mathbf{a}}$, from which the message bits are estimated ($\hat{\mathbf{u}}$). For a system with channel coding, $\hat{\mathbf{a}}$ is used to estimate the interleaved code sequence, $\hat{\mathbf{b}}$, which then deinterleaved ($\hat{\mathbf{b'}} = \mathbf{b}\Pi^{-1}$) and fed to the channel decoder. The channel decoder now comes up with a hard estimate of $\hat{\mathbf{u}}$, denoted as $\hat{\mathbf{u}}$. 

![Figure 1: System model with channel coding.](image-url)
SOFT-DECISION-DPD WITH VITERBI DECODING

The differential detector finds the phase difference at the end of each symbol interval as

\[ \Delta \varphi_k = (\Delta \varphi_k + \eta(t_k) - \eta(t_{k} - T)) \mod 2\pi, \]

for \( k = 0, 1, ..., L - 1 \). In our design, we assume adjacent symbol interference \( (Z = 2) \), this gives [3]

\[ \Delta \varphi_k = a_k \theta_0 + a_{k-1} \theta_1 + a_{k+1} \theta_1 - \theta_{k-1} \]

where

\[ \theta_i = \pi h \int_{iT}^{(i+1)T} g(t) dt \]

(6) implies \( \Delta \varphi_k \) will have one of \( M^3 \) possible values. The phase region between \( 0-2\pi \) is divided into \( R \) sub-regions. The detector then finds the one of the \( R \) possible sub-regions \( (D_k) \), in which \( \Delta \phi_k \) lies. The sequence of phase sub-regions \( D = (D_0, D_1, ..., D_{L-1}) \) is then sent to a branch metric calculator. Let \( \Delta \varphi^l = (\Delta \varphi_0^l, \Delta \varphi_1^l, ..., \Delta \varphi_{L-1}^l) \) be the phase differences corresponding to any transmitted sequence \( \alpha^l = (a_0^l, a_1^l, ..., a_{L-1}^l) \). The branch metric calculator finds the conditional probabilities of receiving \( D \), given \( \Delta \varphi^l \). The metric for the \( l \)-th path in the trellis at a symbol interval \( k \) from [8] is

\[ P(D_k|\Delta \varphi_k^l) = P(\beta_1^l \leq \Delta \varphi_k < \beta_2^l) \]

(7)

\[ = 1 + F(\beta_1^l|\Delta \varphi_k^l) - F(\beta_2^l|\Delta \varphi_k^l), \beta_1^l \leq \Delta \varphi_k^l < \beta_2^l \]

\[ = F(\beta_2^l|\Delta \varphi_k^l) - F(\beta_1^l|\Delta \varphi_k^l), otherwise. \]

\( \beta_1^l \) and \( \beta_2^l \) are the boundaries of the sub-region \( D_k \). Derivations in [13] are modified for GFSK to find \( F \) giving

\[ F(z) = \frac{1}{2\pi} \int_0^{\pi/2} \exp \left[ -\frac{K(P_s + P_d)/P_n}{1 + K + (P_s + P_d)/P_n} E(z, \theta) \right] I(z, \theta) d\theta, \]

\[ E(z, \theta) = \frac{1 - \cos(z) \cos(\theta)}{1 - A \cos(z) \cos(\theta)}. \]

\[ I(z, \theta) = \frac{\sin(z)}{1 - \cos(z) \cos(\theta)} - \frac{A \sin(z)}{1 - A \cos(z) \cos(\theta)}, \]

and,

\[ A = J_0(2\pi f_{dm} T)(P_s + P_d)/P_n \]

\[ = \frac{1 + K + (P_s + P_d)/P_n}{1 - A \cos(z) \cos(\theta)}. \]

\( J_0 \) is the 0th order Bessel function of the first kind, \( f_{dm} \) is the maximum doppler shift (assumed 0 in our simulations). The branch metrics are then used by a \( M^2 \) state Viterbi decoder, to find the ML estimate \( \hat{a} \). The design of nonuniform phase sub-regions for \( M = 2 \) is described in [7], where it is shown that for a given \( R \), carefully selected nonuniform regions perform better than uniform phase regions. Here, since the expected phase differences are closely spaced and the possible phase differences increase exponentially with \( M \), we consider only uniformly spaced phase regions. Simulations (not shown here) reveal that for large enough \( R \), SD-DPD/VD with uniform phase regions performs almost as well as SD-DPD/VD with optimally selected nonuniform regions.

SELECTION OF GFSK PARAMETERS

The power spectral density (PSD) of the GFSK signal is given in [2]. The PSD for the \( M \)-GFSK signal is evaluated at different values of \( h \) and \( B_s T \). From these, we select those values of \( h \) and \( B_s T \) that meet a given spectral efficiency. For a throughput of 5 bps/Hz, we select the GFSK signals that meet the following condition

\[ B_{99n} = \frac{B_{99}}{\log_2(M)} \leq 0.2 \]

Fig. 2 shows the PSD for 2-GFSK at \( h = 0.1 \) and \( B_s T = 0.1 \), which gives \( B_{99n} = 0.172 \). Also shown is the PSD for GMSK \( (h = 0.5, B_s T = \infty \) and \( B_{99n} = 1.18 \)). From the possible values of \( h \) and \( B_s T \) at each \( M \), we select those parameters that give the best tradeoff between error rate and energy efficiency. Since there is no convenient error bound for our system model (to the best of our knowledge), this is achieved through extensive simulations. Fig. 3 shows the BER for uncoded 4-GFSK using SD-DPD/VD, under values of \( h \) and \( B_s T \) that meet the 5 bps/Hz throughput. In all our BER plots, at least 30 frame errors have been generated at each \( E_b/N_0 \). Fig. 3 reveals that selecting \( h = 0.055 \) and \( B_s T = 0.25 \) would result in up to a 10 dB reduction in \( E_b/N_0 \) over other values of \( h \) and \( B_s T \). Similar searches are performed for \( M = 2, 8 \) and 16. The selected set of GFSK parameters are listed in Table I.

COMPARISON WITH NONCOHERENT RECEIVERS

Our proposed receiver is compared with popular noncoherent detectors under identical GFSK parameters. These are the LDID, LDID/VD and DPD/VD. The BER comparison for 8-GFSK (Fig. 4) is presented. This shows the SD-DPD/VD clearly outperforms the other receivers. In fact, it
Comparing with the Coherent Receiver

Coherent ML sequence detection for CPM is described in [2]. The number of states in such a receiver is proportional to $pM^Z$ where, $p = m/h$ and $p$ and $m$ are are relatively prime positive integers. Fig. 5 shows how the coherent receiver compares with the SD-DPD/VD. As expected, when the phase can be perfectly estimated (Coherent: 0 in Fig. 5), the coherent receiver performs better than SD-DPD/VD. However, perfect phase estimation may not be possible or involve considerable complexity. The BER of the coherent receiver is seen to worsen with increasing phase estimation error. In Fig. 5, “Coherent: $3^\circ$” implies the standard deviation in the estimation error is $3^\circ$. In fact, Fig. 5 shows that for estimation errors $> 3^\circ$, the SD-DPD/VD performs better than the coherent receiver at high $E_b/N_o$.

Complexity of the SD-DPD/VD

As seen in the previous section, the number of states in the coherent receiver is proportional to $pM^Z$. It is obvious that the required values of $h$ (Table I) result in large values of $p$. As an example, at $M = 16$, $h = 0.05$, which gives $p = 100$. The coherent receiver will hence have 25600 states (limiting $Z$ to 2). In addition, estimating the phase would further add to the complexity, making such a receiver unfeasible for our application. The SD-DPD/VD in contrast has $M^2$ states and being noncoherent does not need phase information at the receiver.

SD-DPD/VD with Convolutional Coding

SD-DPD/VD with convolutional coding was studied in [8] and coding gains up to 10 dB were reported. This gain is however obtained at the expense of bandwidth. Convolutional coding without bandwidth expansion has been studied in [12, 14, 15, 16, 17]. As done in [12], the PSD is first found for the uncoded GFSK signal ($S_x(f)$) and then the PSD for the convolutional coded GFSK signal is found by $S'_x(f) = R_cS_x(R_cf)$, where
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
BER
Figure 6: BER comparison of uncoded and rate 1/2 convolutional coded 4-GFSK at $K = 3$ dB with SD-DPD/VD ($R = 80$).


