Joint Power Allocation and Relay Selection for Multiuser Cooperative Communication

Kanchan Vardhe, Associate Member, IEEE, Daryl Reynolds, Member, IEEE, and Brian D. Woerner, Senior Member, IEEE

Abstract—User cooperation, whereby multiple users share their antennas and transmit to a common destination in a collaborative manner, has been shown to be an effective way to achieve spatial diversity. We propose in this paper, a strategy to minimize the total transmit power in a decode-and-forward (DF) multi-user, multi-relay cooperative uplink, such that each user satisfies its quality-of-service (QoS) data rate. Each user in the proposed system transmits its own data towards the base station and also serves as a relay for other users. The base station assigns one or more relays to each user in order to minimize total power in the uplink. The relay selection is based upon the instantaneous user to base station channels, inter-user channels and also the target rates of the users. The simulation results indicate significant power savings over a non-cooperative uplink, under proposed joint relay selection and power minimization algorithm in a DF cooperative uplink when using a space-time coded cooperative diversity.

Index Terms—cooperative diversity, mutual information, power allocation.

I. INTRODUCTION

In those wireless applications where it is impractical to implement multiple antennas at the mobile units, user cooperation seems to be a viable option to achieve spatial diversity [1]. User cooperation provides diversity gains using antennas of neighboring users in the network. We present in this paper, an algorithm that performs relay selection while minimizing the total transmit power and satisfying certain quality-of-service (QoS) requirement in multi-user decode-and-forward (DF) cooperative networks.

Recently, references [2], [3] indicated performance improvements due to use of cooperative diversity over point-to-point links in wireless networks when using single antenna at mobile nodes and equal power allocation. Later, efforts were made to further improve the performance of cooperative diversity by optimal power allocation and optimal group assignment. For example, considering a three terminal DF relay terminal network, optimal power allocation was studied when optimizing either achievable rates [4], outage events [5], or outage probability [6]. Using outage probability as an optimization criterion and total power constraint, the authors in [7] optimally allocated relay powers in DF multi-hop wireless networks. The power control algorithm, which attempts to minimize the outage probability under short-term and long-term total power constraints, was studied in [8] for two-user cooperation scheme, while [9] considered optimization of high-SNR approximations of outage probability for the multi-user space-time coded DF protocol. In [10], an opportunistic DF protocol was developed where a relay terminal is utilized depending on the overall network state with dynamic allocation of time and power. The above work demonstrated a significant performance improvement due to optimal power allocation over equal power allocation in cooperative networks. The partner choice problem in a DF cooperative network was investigated in [11], where authors devised a method to choose a single partner among available partners to increase the user cooperation gain. Grouping schemes for regenerative cooperative network of N nodes, based on both centralized and distributed control strategies were presented in [12]. Bletsas et al., [13] proposed the best relay selection method that takes into account the instantaneous channel conditions of both source to relay and relay to destination channels.

Most previous work on cooperative diversity either 1) makes no attempt to optimize power, or 2) optimizes power, assuming a cooperating group has been assigned a priori. There has been very little work on joint relay selection and power allocation in multi-user cooperative networks. Power allocation for space-time coded DF cooperative diversity was studied in [14], where the authors presented a suboptimal solution to minimizing the outage probability where the source power is fixed (perhaps fixing the decoding set in effect) and the remaining power is equally distributed among the relays. The sub-optimal source power is obtained numerically through exhaustive search. Relative to [14], the novelty of our approach is the non-suboptimal solution for the source and relay powers and joint relay selection as explained in the sequel.

We consider a user-cooperative uplink where users have been allocated orthogonal channels for transmission (using, for example, orthogonal CDMA spreading codes1). Each user has its own data to send to the base station. We develop in this paper, a strategy to minimize the total transmit power in a decode-and-forward (DF) user cooperative uplink, such that each user satisfies its quality-of-service (QoS), data rate.

1Non-orthogonal channels can be dealt with using decorrelating multuser detection and an additional noise variance factor [15].
We model the total power minimization problem as an optimization problem where the objective function (total network power) is a convex function of user powers and the constraints are target rates of users which are concave functions. We then solve the optimization problem by Lagrange multiplier method. The solution to the optimization problem in DF cooperative uplink leads to an iterative algorithm that jointly performs relay selection for cooperation and optimally allocates source and the relay powers.

The remainder of the paper is organized as follows. Section II introduces the uplink system model, channel model, and also describes the user cooperation protocol. Section III considers power consumption under non-cooperative uplink as a baseline for the proposed power minimization algorithm. Section IV describes the proposed joint relay and power allocation algorithm under both diversity combining and code combining. Simulation results are presented in Section V and Section VI concludes.

II. SYSTEM MODEL

The uplink consists of $K$ users that have been allocated orthogonal channels. Each user has its own data to transmit to the base station, potentially using other users as relays. We consider a decode-and-forward (DF) protocol that consists of two transmission phases. During the first phase, each user $k$ broadcasts its message to the base station with power $p_k$. In the second phase, other users that can decode the $k$-th user’s transmission form a decoding set $D(k)$ and may serve as relays. Based on our proposed relay selection criterion, some relays from the decoding set would, however, remain silent even if they can decode the $k$-th user’s transmission in order to reduce the power consumption in the network. Since each user acts as a source during the first time phase and may serve as relay during second time phase, we use the terms user, source and relay interchangeably. We consider two practical scenarios. In the first case, selected relays could transmit using incremental redundancy which leads to code combining of the relayed information. As an alternative, the selected relays may use a distributed space-time code for the source’s transmission that leads to diversity combining of the relayed transmissions [3]. Incremental redundancy type of cooperation protocol enjoys full spatial diversity gains but at the expense of bandwidth inefficiency since each user requires a separate orthogonal channel for its own transmission and for relaying other user’s data. More bandwidth efficient space-time coded cooperation provides full spatial diversity gains, however, requires symbol level inter-user synchronization.

The base station is assumed to have the knowledge of all instantaneous channel conditions including user to base station channels as well as inter-user channels and makes all assignment decisions2. It then conveys the relay assignment and the optimized powers to users through a low rate feedback channel. The user-to-base station channels and the inter-user channels undergo independent quasi-static Rayleigh fading and path loss. The inter-user channels are non-symmetric, i.e., the channel between user $i$ and $j$ is, in general, different from the channel between user $j$ and $i$. All channels are statistically modeled as zero-mean, circularly symmetric complex Gaussian random variables. We model distance dependent path loss without shadowing. The channel (distance) between the $k$-th user and the base station is denoted by $h_{k,d} (d_{k,d})$ while $h_{i,j}$ ($d_{i,j}$) denotes the inter-user channel (distance) between users $i$ and $j$. Let $g_{i,j}$ denote the channel gain for the link between users $i$ and $j$ where $g_{i,j} = |h_{i,j}|^2/d_{i,j}^{\alpha}$, $\alpha$ being the path loss coefficient.

III. POWER CONSUMPTION IN A NON-COOPERATIVE UPLINK

For certain traffic types, e.g., real-time video, it is necessary that the transmission meets certain QoS requirement, e.g., data rate. The total network power consumption under a non-cooperative scenario, where users expend power to achieve target rates, can be found in a straightforward manner as follows.

The signal received by the base station, $d$ due to the $k$-th user’s transmission is

$$r_d[n] = \sqrt{p_k} h_{k,d} x_k[n] + v[n]$$

(1)

where $p_k$ is the power used by the $k$-th source and $v[n]$ is the receiver noise and is distributed as $v[n] \sim N_c(0, 1)$. The mutual information for the channel between the $k$-th user and the base station is

$$\log (1 + p_k g_{k,d}).$$

(2)

The minimum transmit power required at the source $k$ to achieve the target rate $R$, under no cooperation, is

$$p_{k,nc} = \frac{2R - 1}{g_{k,d}}.$$ 

(3)

The total uplink power under no cooperation scenario is

$$p_{nc} = \sum_{k=1}^{K} p_{k,nc}.$$ 

(4)

IV. POWER MINIMIZATION IN A USER-COOPERATIVE UPLINK

In wireless networks, at any given time instant, users might experience very different fading channel conditions. Users experiencing deep fades will then have to expend large amount of power in order to meet the QoS constraints as can be seen from (3). Spatial diversity created due to user cooperation might reduce the probability of deep fades, thus reducing the total transmit power. To further enhance the performance of user-cooperative uplink, it is imperative to devise algorithms for optimal relay selection and power allocation across source and relay terminals. Optimal power allocation in a DF uplink is complicated by the fact that decoding set is a function of both inter-user channels as well as the source power in the first phase. Given all instantaneous channel conditions, one of the naive approaches to the relay selection would be to

\footnote{The assumption on the knowledge of all instantaneous channel gains at the base station is not practical. However, the results provide baseline for comparison with practical systems and also provide guidance in the design of practical systems.}
perform exhaustive search over all possible decoding sets for a first phase and then deciding upon the decoding set that minimizes the total uplink transmit power after optimal power allocation. This relay selection process would alone require \( \sum_{i=1}^{K-1} {K-1 \choose i} \) iterations per user, which is impractical for larger number of uplink users.

We develop an iterative algorithm that jointly performs relay selection for the purposes of cooperation and optimally allocates source and the relay powers. This iterative method requires only up to \( K \) iterations. In this section, we state the proposed power minimization problem with respect to general mutual information expressions. In the later sections, we consider specific cooperative diversity protocols along with corresponding mutual information expressions and then provide solution to the joint relay selection and power minimization problem. The proposed power minimization problem can be formulated as follows.

\[
\text{minimize} \sum_{k=1}^{K} \left\{ p_k + \sum_{r=1}^{K} p_{k,r} \right\} \tag{5}
\]

subject to

\[
\begin{align*}
I_1 &= R_1 \\
I_2 &= R_2 \\
\vdots \\
I_K &= R_K \\
p_k &\geq p_{k,\min}; \quad k = 1, 2, \ldots, K \\
p_{k,r} &\geq 0; \quad k = 1, \ldots, K; \\
r = 1, \ldots, K, r \neq k
\end{align*}
\]

where \( p_k \) is the \( k \)-th user’s transmit power during phase I, \( p_{k,r} \) is the transmit power by relay \( r \) when forwarding \( k \)-th user’s message, \( I_k \) is the mutual information for the channel between user \( k \) and the base station which will be defined in subsequent sections, \( R_k \) is the target rate of user \( k \), and \( p_{k,\min} \) is the minimum power that source \( k \) transmits with during the first time phase that helps choose relays. The power \( p_{k,\min} \) is updated in each iteration of the proposed iterative power minimization algorithm in order to select the most efficient (optimal) set of relays from the decoding set for source’s transmission. The role of \( p_{k,\min} \) will be clarified further in the next Section. In the discussion to follow, we assume for the sake of exposition that the target rates of all users are the same, i.e., \( R_1 = R_2 = \cdots = R \).

In the above optimization problem, the minimization is done over source and the relay powers. The objective function here is an affine function of source and relay powers and hence is a convex function. Each constraint function, i.e., the target rate is a concave function. The convex optimization problem can now be solved by Lagrange multiplier technique [17]. The solution to the above problem when selecting the optimal set of relays from a decoding set leads to an iterative algorithm as will be explained in the sequel.

A. Power Minimization under Code Combining

In this section, we develop a power optimization protocol for the incremental redundancy type coded cooperation protocol discussed in [3]. During the first time phase, each user \( k \) transmits to the base station. During the second time phase, each relay selected after executing the proposed power minimization algorithm transmits the source’s information over orthogonal subchannels. Each relay has it’s own unique codebook for its own data and for each potential source’s data. As an example, each relay could transmit a different part of the codeword which results in a code combining at the base station. Similar to the non-cooperative case, the mutual information due to the \( k \)-th user’s transmission during the first phase is

\[
\frac{1}{K} \log \left(1 + p_k g_k,d \right). \tag{6}
\]

The factor \( 1/K \) is due to the fact that each source transmits during \( 1/K \) of total time slots in incremental redundancy-based coded cooperative diversity [3]. A potential relay will be able to decode \( k \)-th user’s message if the realized mutual information between user \( k \) and the relay \( r \) is greater than the fixed target spectral efficiency \( R \). Therefore, the relay will be in the decoding set of user \( k \) if

\[
\frac{1}{K} \log \left(1 + p_k g_k,r \right) \geq R, \tag{7}
\]

i.e.,

\[
p_k \geq \frac{2^{KR} - 1}{g_k,r}. \tag{8}
\]

We denote by \( p_{k,r,\min} \), the minimum power source \( k \) should transmit with that will guarantee successful decoding at the relay \( r \). Hence

\[
p_{k,r,\min} = \frac{2^{KR} - 1}{g_k,r}. \tag{9}
\]

The overall average mutual information between user \( k \) and the base station under code combining is \(^3\)

\[
I_{k,cc} = \frac{1}{K} \log \left(1 + p_k g_k,d \right) + \frac{1}{K} \sum_{r=1}^{K} \mathbb{1}_{p_{k,r} > p_{k,r,\min}} \log \left(1 + p_k,r g_r,d \right) \tag{10}
\]

where \( \mathbb{1}_{x>y} \) is a indicator function

\[
\mathbb{1}_{x>y} = \begin{cases} 1, & \text{if } x > y \\ 0, & \text{otherwise} \end{cases} \tag{11}
\]

Now with mutual information \( I_k \)’s defined for the special case of code combining, we return to the optimization problem

\(^3\)Although it appears that spectral efficiency under incremental redundancy cooperative diversity goes to zero as \( K \) goes to infinity, following the technique in [16] it can be easily shown that it approaches a fixed non-zero value.
in (5). The Lagrangian equation for the optimization problem in (5) is

$$\sum_{k=1}^{K} \left\{ p_k + \sum_{r=1}^{K} p_{k,r} \right\} - \sum_{i=1}^{K} \lambda_i (I_{i,cc} - R_i) = 0. \quad (12)$$

By taking the derivative of (12) with respect to the source and relay powers, applying the Kuhn-Tucker conditions, and taking into account the non-negativity constraints,

$$p_k = \max \left( \frac{\lambda_k}{\log 2} - \frac{1}{g_{k,d}} p_{k,\min} \right) ; \quad k = 1, \ldots, K \quad (13)$$

$$p_{k,r} = \frac{1}{p_k > p_{k,r,\min}} \times \max \left( \frac{\lambda_k}{\log 2} - \frac{1}{g_{r,d}} 0 \right)$$

for \( k = 1, \ldots, K; r = 1, \ldots, K; r \neq k \) \quad (14)

where the powers \( p_{k,\min} \) in the first iteration are set according to the following rule:

$$p_{k,\min} = \begin{cases} p_{k,nc}, & \text{if } p_{k,r,\min} \geq p_{k,nc}, \forall r; \\ \arg \max_r \Omega, & \text{otherwise} \end{cases} \quad (15)$$

where \( \Omega = \{ p_{k,r,\min} | p_{k,r,\min} \leq p_{k,nc} \} \). Equation (15) has the following interpretation. During the first iteration, if the source to destination channel is stronger than any of the source to (potential) relay channels, then only direct transmission is preferred, else the source transmits with a minimum power that guarantees largest possible decoding set for its transmission. Hence, at the start of the first iteration of the algorithm, all potential relays for the source \( k \) for which \( p_{k,\min} \geq p_{k,r,\min} \) are the decoding relays. \( \lambda_k \) is found by substituting source powers \( p_k \) and the relay powers \( p_{k,r} \) from (13) and (14) in the \( k \)-th constraint of (5) and solving the transcendental equation in \( \lambda_k \). The source and relay powers are then obtained by substituting for \( \lambda_k \) in (13) and (14). From the set of decoding relays considered during the previous iteration, the relays that resulted in corresponding \( p_{k,r} = 0 \) after power minimization, are excluded from a set of decoding relays. This is because for any \( p_{k,r} = 0 \), the resulting power minimization suggests not selecting that particular relay for cooperation purposes. The corresponding minimum source power to have second largest possible decoding set is then updated using equation (15) as also the source and relay powers. A total of up to \( K \) iterations are needed to find the most efficient set of relays and the corresponding relay powers for each source. If the computed transmit powers do not change between successive iterations, the iterative procedure described in the proposed algorithm can be stopped. In a conventional DF protocol using constant power allocation, the relays remain silent if they cannot decode the source’s transmission. However, in the proposed setup, whenever it is advantageous for the source to utilize a relay, it transmits with a sufficient power level that guarantees successful decoding at the relay. This also helps in finding the optimal source power for the first phase of transmission.

**B. Power Minimization under Diversity Combining**

We consider here a space-time coded protocol where during the second time phase of cooperation, the selected relays from the decoding set of a particular user use an ideal space-time code and hence can transmit simultaneously on the same subchannel [3]. We develop an iterative relay selection and power minimization algorithm very similar to the code combining case discussed earlier. The mutual information due to \( k \)-th user’s transmission during the first phase is

$$\frac{1}{2} \log (1 + p_k g_{k,d}). \quad (16)$$

The factor \( 1/2 \) is due to the time phase orthogonality in space-time coded protocol. A potential relay will be able to decode \( k \)-th users message if the realized mutual information between user \( k \) and the relay \( r \) is greater than the fixed spectral efficiency \( R \). Therefore, relay will be in the decoding set of user \( k \) if

$$\frac{1}{2} \log (1 + p_k g_{k,r}) \geq R, \quad (17)$$

i.e.,

$$p_k \geq \frac{2^R - 1}{g_{k,r}}. \quad (18)$$

Hence

$$p_{k,r,\min} = \frac{2^R - 1}{g_{k,r}}. \quad (19)$$

The overall average mutual information between user \( k \) and the base station under diversity combining is

$$I_{k,sc} = \frac{1}{2} \log \left( 1 + p_k g_{k,d} \right) + \frac{1}{2} \log \left( 1 + \sum_{r=1}^{K} p_{k,r,\min} p_{k,r} g_{r,d} \right). \quad (20)$$

The diversity combining case with space-time coded protocol thus differs from the code combining case in the bandwidth utilization factor in front of the log terms which is \( 1/K \) for the incremental redundancy based coded cooperative diversity and \( 1/2 \) for the space-time coded cooperative diversity. It also differs in the mutual information expressions in that incremental redundancy based cooperative diversity with code combining involves sum-log expression while space-time coded diversity with diversity combining involves log-sum expressions for the second phase of transmission. The Lagrangian equation for the optimization problem in (5) is

$$\sum_{k=1}^{K} \left\{ p_k + \sum_{r=1}^{K} p_{k,r} \right\} - \sum_{i=1}^{K} \lambda_i (I_{i,sc} - R_i) = 0. \quad (21)$$

By taking the derivative of (21) with respect to the source and relay powers, applying the Kuhn-Tucker conditions and taking into account the non-negativity constraints,
\[ p_k = \max \left( \frac{\lambda_k}{\log 2} - \frac{1}{g_{k,d}}p_{k,\text{min}} \right) : k = 1, \ldots, K \]  

\[ p_{k,r} = \max \left( \frac{\lambda_k}{\log 2} - \frac{1}{g_{r,d}} - \sum_{i=1 \atop i \neq r}^{K} \mathbb{P}(p_k > p_{k,\text{min}}) \times \frac{p_{k,i}g_{k,i}}{g_{r,d}} \right) \]

for \( k = 1, \ldots, K; r = 1, \ldots, K; r \neq k \). (23)

As seen from (23), the relay powers are now interdependent. The relay powers \( p_{k,r} \) are found sequentially and in an iterative fashion, where sequence order is not crucial to finding the optimum solution. The relay powers are initially set to zero and are updated as the sequence of relay power equations in (23) is traced, for a particular value of \( \lambda_k \). By varying the value of \( \lambda_k \), a transcendental equation in \( \lambda_k \) is solved. The remaining iterative steps of the underlined joint relay selection and power allocation algorithm remain the same as the code combining case.

V. SIMULATION RESULTS

We assume for the simulation purposes that the users are distributed uniformly over a grid of 1 \( \times \) 1 units with the base station located at position (1,1). All channels including the inter-user channels and the user-to-base station channels are independent. The channel coefficients are complex Gaussian with zero mean and unit variance. The path loss coefficient \( \alpha \) is set to 3.

Fig. 1 indicates the average power consumption in a uplink with respect to total number of users under direct transmission and two different user cooperation scenarios. The target rate is \( R = 1 \) bit/sec/Hz. It is seen that for a fixed rate, as we increase the total number of uplink users, the average total uplink power consumption under incremental redundancy-based cooperative diversity with code combining exceeds that of direct transmission. We see that incremental redundancy-based cooperation improves performance up to about 5 users. The space-time coded protocol with diversity combining performs uniformly better than no cooperation and incremental redundancy-based cooperation with code combining. This is because, with an increase in the total number of users, each potential relay requires a separate orthogonal subchannel in incremental redundancy-based cooperative diversity, hence, making the system bandwidth inefficient. This loss in the spectral efficiency in case of incremental redundancy based cooperation dominates any gain due to cooperation. The system utilizing a space-time coded protocol, however, requires all relays to transmit over the same subchannel and is hence bandwidth efficient when compared to incremental redundancy-based protocol. Therefore, the space-time coded protocol offers significant power savings over no cooperation (direct transmission) under the proposed relay selection and power minimization algorithm.

Fig. 2 illustrates the average power consumption in a user cooperative uplink and under direct transmission, as a function of target rate on a logarithmic scale. It is observed that incremental redundancy-based cooperative diversity with code combining is better than no cooperation up to target rate of 2 bit/Hz/sec for 3 users and up to target rate of 1 bit/sec/Hz for 5 users. The figure indicates that for fewer number of users and target rates of interest, the average total power consumption under both incremental redundancy-based and space-time coded cooperative diversity is significantly less than the direct transmission. For higher target rates and more total users in the uplink, the space-time coded protocol outperforms both direct transmission and incremental redundancy-based cooperative diversity, in terms of power consumption.
VI. CONCLUSION

In this paper, we propose a strategy to minimize the total uplink transmit power in a decode-and-forward (DF) user cooperative uplink such that each user satisfies its target data rate. The proposed iterative algorithm for minimizing the total uplink power jointly performs relay selection for the purposes of cooperation and optimally allocates source and the relay powers. We develop a power minimization scheme for incremental redundancy-based cooperative diversity with code combining (of relayed information) and space-time coded protocol with diversity combining. With respect to incremental redundancy-based cooperative diversity, we find that the cooperation is beneficial in terms of minimizing the total uplink power at lower target rates and less number of cooperating users. Significant cooperation gains could be obtained using a space-time coded cooperative diversity protocol over the wide range of target rates and total number of users when using the proposed joint relay selection and power minimization algorithm. A practical approach to network power minimization based on the average channel gains and average rates remains a topic of future work.

REFERENCES