1 Instructions

1. The homework is due on September 15, in class. Each question is worth 4 points.
2. You may assume that all function placeholders are monotonically non-decreasing.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Given an array $A$ of $n$ integer elements, how would you find the second smallest element in $n + \log_2 n$ comparisons.

2. Indicate whether each of the following identities is true or false, giving a proof if true and a counterexample otherwise.
   
   (a) $f(n) + o(f(n)) \in \Theta(f(n))$.
   (b) $(f(n) \in O(g(n))) \land (g(n) \in O(h(n))) \Rightarrow (f(n) \in O(h(n)))$.
   (c) $\log^{1/\epsilon} n \in O(n^\epsilon), \ (\forall \epsilon) \ 0 < \epsilon < 1$.
   (d) $2^n \in \Omega(5^{\log_2 n})$.

3. Devise a Divide-and-Conquer procedure for computing the $k^{th}$ largest element in an array of integers. Analyze the asymptotic time complexity of your algorithm. (Hint: Use the Partition procedure discussed in class.)

4. Argue the correctness of the MERGE() procedure discussed in class. (Hint: Write a recursive version of MERGE() and then use induction.)

5. What is the value returned by Algorithm (2.1) when called with $n = 10$?

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Function LOOP-COUNTER(n)
1: count = 0
2: for (i = 1 to n) do
3:     for (j = 1 to i) do
4:         for (k = 1 to j) do
5:             count ++
6:     end for
7: end for
8: end for
9: return(count)
```

Algorithm 2.1: Loop Counter