Advanced Analysis of Algorithms - Homework III

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1 Instructions

1. The homework is due on November 17, in class. Each question is worth 4 points.
2. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Is there a problem in the complexity class \( P \), such that all problems in \( P \) can be polynomially transformed to this problem?
2. Show that a language \( L \) can be verified in deterministic polynomial time if and only if it can be decided by a non-deterministic algorithm in polynomial time.
3. Design a backtracking algorithm for the 3SAT problem.
4. Consider an instance of the Subset-Sum problem, where \( S = \{2, 10, 13, 17, 22, 42\} \) and \( B = 52 \). Solve this instance using backtracking, showing all the steps.
5. Consider the following graph coloring algorithm for coloring the vertices of a graph using the fewest number of colors:

   **Function** FIND-OPTIMAL-COLOR(\( G=<V,E> \))
   
   1: Let \( V_{un} = V \) and \( C_u = \{1, 2, \ldots, n\} \).
   2: while \( V_{un} \neq \emptyset \) do
   3: \( c_{cur} \) is the smallest indexed color in \( C \).
   4: Assign \( c_{cur} \) to as many vertices as possible in \( V_{un} \) making sure that a vertex with index number \( k \) is considered before a vertex with index number \( k + 1 \).
   5: Delete all the colored vertices from \( V_{un} \).
   6: Delete \( c_{cur} \) from \( C \).
   7: end while

   **Algorithm 2.1**: Graph Coloring Algorithm

   \( V_{un} \) is the set of uncolored vertices and \( C_u \) is the set of unassigned colors.

   Is Algorithm (2.1) optimal? Justify your answer with a proof or a counterexample.