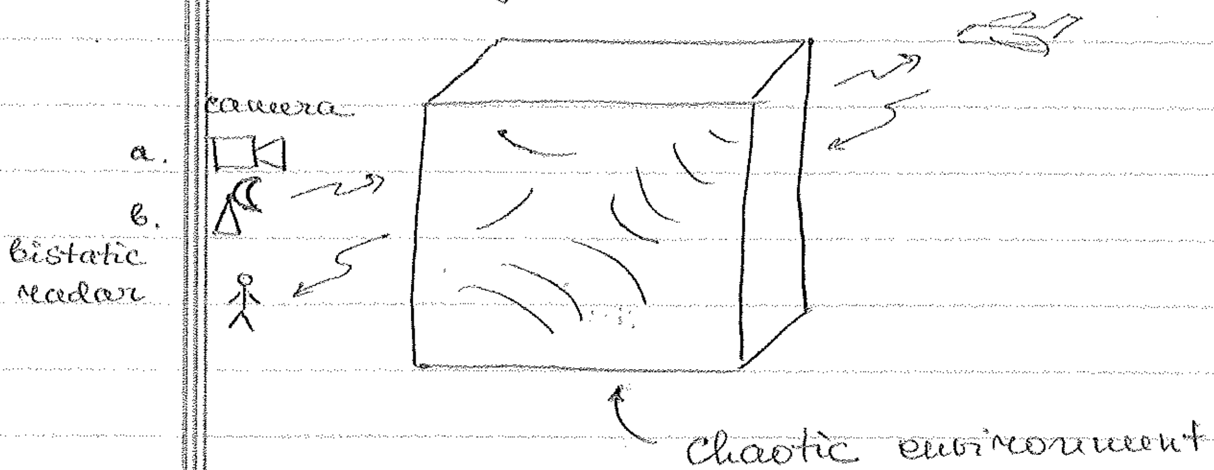
Read Ch. 1
Leon-Garcia

EE 513
Fall 2008

I. Probability Models



Designers have to design system that operate reliably in chaotic environment

ex. • recognition systems (pattern, object, biometric) have to be able recognize patterns with small error

Attenuation, Distortion, Scattering, Fading • wireless sensor networks are required to operate reliably when installed in cities for monitoring activities or detect some phenomena.

• power networks (electric networks) have to deal with certain fluctuations of load (not always predictable)

Design of these type of systems often relies on probability models.

Design Steps:

Select a cost function (criterion or design metric) to be optimized.
↓
physical model { Come up with explanation of data (what generated data)

↓

Mathematical Model { Come up with a set of mathematical equations describing physical model

Has to be relatively simple

Has to describe physics well

↓

Test mathematical model (use simulations, testing data)

Model describes (predicts) data well (cost fnct.)

Implement model in practice

Model does not explain data well (cost function)

Go back to physical model. Modify it.

Model – approximate representation of underlying physics
(ex. Newton's laws explain behavior of moving object)

Mathematical Model – set of rules and variables describing physical model.

<u>Deterministic model</u> – solution of set of equations specifies outcome.
Outcomes of experiment are reproduciable with each experiment.

(ex. Based on Newton's law

$$\underset{\uparrow \text{ known}}{m}\ddot{x} = \underbrace{\sum_i \text{forces}}_{\text{if forces are known and the same}}$$

$\Rightarrow \ddot{x}$ is the same in each experiment)

Probability Models

In many cases in practice outcomes of experiment (under the same set up) are not reproduciable.
These are random experiments.
Deterministic models do not describe them well.

(ex. Toss a coin.
Possible outcomes $\{H, T\}$.
In each experiment we cannot ensure that H will be observed.)

H, T are outcomes.
$\{H, T\} = S$ a set of possible outcomes called sample space.

The theory of probability deals with averages of mass phenomena (simultaneous or sequential).

Observation: In many applications, certain averages approach a constant value.

ex. Toss a coin n times. Count # "heads."
Let n_H be the # "heads" observed.

$$\frac{n_H}{n} \to 0.5 \text{ as } n \to +\infty \qquad (*)$$

This property is called statistical regularity.
The left side of (*) — relative frequency.
The right side of (*) — probability of "heads".

Properties of relative frequency :

1. Since $0 \le n_H \le n$, the relative frequency

$$0 \le \frac{n_H}{n} \le 1 .$$

2. For cumulative events such as " Even number of dots appear on the surface of die "

$$f_E(n) = \frac{n_2 + n_4 + n_6}{n} = f_2(n) + f_4(n) + f_6(n)$$

relative frequency of E = event

Note : the events "2", "4", "6" do not occur simultaneously. They are exclusive – if one occurs, the other cannot occur simultaneously.

These properties provide basis for axiomatic approach to probability theory (due to Markov).

It assumes :

possible elementary outcomes are identified → 1. A random experiment is described

2. Events on elementary outcomes and their combinations are specified (think of Algebra on outcomes and formed events)

3. Events are assigned a weight called probability of event.

Probability of event E has to satisfy

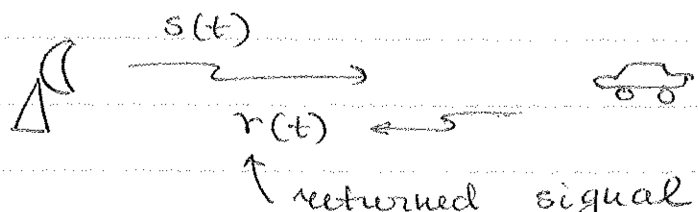
1. $0 \le P[E] \le 1$
2. $P[S] = 1$
3. If two or more events are mutually exclusive (do not occur simultaneously),

$$P[E_1 \cup E_2] = P[E_1] + P[E_2]$$

How to <u>build</u> Probability Model?

(follow 3 steps on p. 4)

<u>ex.</u> (Radar Imaging)



<u>Physics</u>:

- <u>EM waves</u> (usually modulated short pulse)
- <u>Grainy</u> surface of object

Mathematical model: scene of reflections is Gaussian RV (Central Limit Theorem)

Processing unit evaluates energy reflected from objects at times $t_1, t_2, \ldots, t_n$ and stores in a vector.

The returned sensed signal has noise in it due to the receiver. A typical model for received vector $\underline{r}$ is

(HRR) = high resolution radar

$$\underline{r} = \underline{s}_{\text{reflected}} + \underline{w}$$

$\underline{s}_{\text{reflected}}$: $CN\left(\underline{0}, \begin{bmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_n^2 \end{bmatrix}\right)$ due to cumulative effect (CLT)

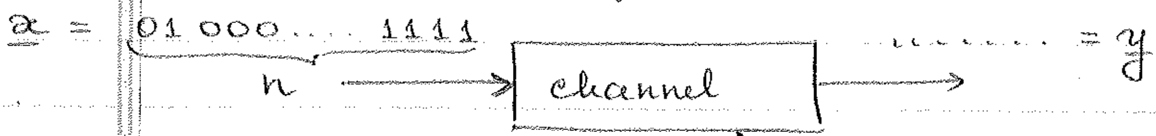
$\underline{w}$: complex valued white noise with psd $\frac{N_0}{2}$

$\underline{s}_r$ and $\underline{w}$ are independent

Therefore,

$$\underline{r} \sim CN\left(\underline{0}, \begin{bmatrix} \sigma_1^2 + \frac{N_0}{2} & & \emptyset \\ & \ddots & \\ \emptyset & & \sigma_n^2 + \frac{N_0}{2} \end{bmatrix}\right)$$

ex. (Binary asymmetric channel)

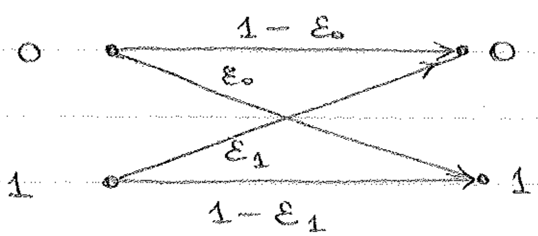


Signals are communicated to the receiver end in the form of sequences of 0s and 1s.

adds errors, distorts

If bits are transmitted individually and channel does not have memory, then bits in y are also treated individually.

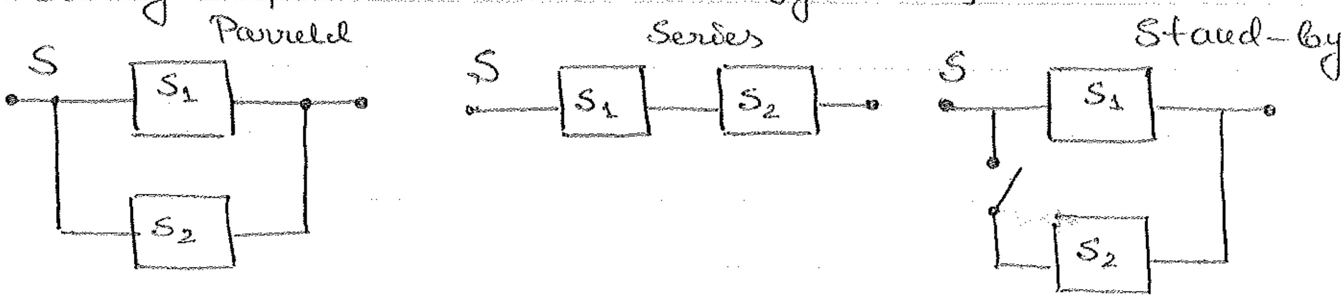
One of typical models adopted for channel is BAC, where "0"s and "1"s are treated differently. For example, the error of communicating 1 is higher than the error of communicating 0.



Noise is binary in this case and added "modulo 2"

ex. (System reliability)

We know reliability of each individual system and would like to evaluate reliability of a combined systems



A typical measure of reliability is time to failure. Let x, y, and z be times to failure for S_1, S_2, and S.

Parallel:

S fails if both S_1 and S_2 fail.
Time to failure for S is

$$z = \max(x, y).$$

Series:

S fails if either S_1 or S_2 fail.
Time to failure for S is

$$z = \min(x, y).$$

Stand-by:

S fails when S_2 fails, that is, S_2 is in reserve while S_1 operates. Once S_1 fails, S_2 operates. Time to failure for S is

$$z = x + y.$$

Time to failure is large at the origin (at the starting point of experiment) and exponentially decreasing in the process of experiment, as the time evolves.