Homework 2

CS 591Q/791V - Pattern Recognition
Instructor: Dr. Arun Ross
Due Date: March 12, 2009

Note: You are permitted to discuss the following questions with others in the class. However, you *must* write up your *own* solutions to these questions. Any indication to the contrary will be considered an act of academic dishonesty. Code developed as part of this assignment should be placed in a zip file and sent to arun.ross at mail.wvu.edu with the subject line "CS 591Q/791V : Homework 2".

---

1. [10 points] The Poisson distribution for a discrete variable $x = 0, 1, 2, \ldots$ and real parameter λ is:
$$P(x|\lambda) = e^{-\lambda}\frac{\lambda^x}{x!}.$$
Consider two equally probable classes C_1 and C_2 having Poisson distributions but with different parameters λ_1 and λ_2, respectively. For definiteness assume $\lambda_1 > \lambda_2$. What is the Bayes classification rule?

2. [15 points] Consider the following class-conditional densities for a three-category problem involving two-dimensional features:
$p(\boldsymbol{x}|C_1) \sim N\left((0,0)^t, I\right);$
$p(\boldsymbol{x}|C_2) \sim N\left((1,1)^t, I\right);$
$p(\boldsymbol{x}|C_3) \sim \frac{1}{2}N\left((0.5,0.5)^t, I\right) + \frac{1}{2}N\left((-0.5,0.5)^t, I\right).$
By explicit calculation of the *posterior* probabilities, classify the two-dimensional point $\boldsymbol{x} = (0.3, 0.3)^t$ based on the Bayes decision rule.

3. [5 points] Consider the three-dimensional normal distribution $p(\boldsymbol{x}) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu} = (1,2,2)^t$ and $\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 2 \\ 0 & 2 & 5 \end{bmatrix}$. What is $p([0.5, 0, 1]^t)$.

4. [20 points] Consider a two-category classification problem involving two-dimensional feature vectors of the form $\boldsymbol{x} = (x_1, x_2)^t$. The two categories are C_1 and C_2, and
$p(\boldsymbol{x} \mid C_1) \sim N\left((0,0)^t, I\right),$
$p(\boldsymbol{x} \mid C_2) \sim N\left((1,1)^t, I\right),$
$P(C_1) = P(C_2) = \frac{1}{2}.$

(a) Calculate the Bayes decision boundary and write down the Bayes decision rule.

(b) Generate $n = 100$ patterns from *each* of the two class-conditional densities and plot them in a two-dimensional feature space. Draw the Bayes decision boundary computed in (a) on this plot. What is the empirical error rate when classifying the generated patterns using the Bayes decision rule.

(c) Repeat (b) above by varying the value of n from 100 to 1000 in steps of 100. Also, plot the empirical error rate as a function of n.

5. Consider a two-category (ω_1 and ω_2) classification problem with equal priors. Each feature is a two-dimensional vector $\boldsymbol{X} = [x_1, x_2]^t$. The class-conditional densities are:
$p(\boldsymbol{X}|C_1) \sim N(\mu_1 = [0, 0]^t, \Sigma_1 = 2I)$,
$p(\boldsymbol{X}|C_2) \sim N(\mu_2 = [1, 2]^t, \Sigma_2 = I)$.

(a) [15 points] Generate 100 bivariate *random* training samples from each of the two densities. Find the maximum likelihood estimates of μ_1, μ_2, Σ_1, and Σ_2 using these training samples.

(b) [10 points] Compute the Bayes decision boundary using the *estimated* parameters and plot it along with the training points.

(c) [10 points] In the same graph, draw the Bayes decision boundary when the *true* parameters are known.

Note: Use the "mvnrnd" (multivariate normal random point generator), "plot" and "ezplot" commands in MATLAB to generate random data, plot the data, and view the decision boundary.
