2. Review of Set Theory

CS 336

Lecture 2

Thursday,August 24, 2000

Outline

- Logic
- Sets
- Relations and Functions

Logic

Some Logical Operators

p	q	p & q (and)
F	F	F
F	T	F
T	F	F
T	T	T

p	q	p or q
F	F	F
F	T	T
T	F	T
T	T	T

p	not p
F	T
T	F

Some More Logical Operators

p	q	p => q (implies)
F	F	T
F	T	T
T	F	F
T	T	T

p	q	p <=> q (equivalence)
F	F	T
F	T	F
T	F	F
T	T	T

Logical Quantifiers

"there exists"

ex. There exists an integer n such that $4 < n < 10$

"there exists uniquely"

ex. There exists a unique integer n such that $4 < n < 6$

"for all"

ex. For all integer n, $n + n$ is an integer

ex. For all integer n, $n + 0 = n$

Sets

Subsets

X is a subset of Y <=> for all z in X, z is in Y

Two sets are equal iff they have the same elements.

- X = Y <=> X is a subset of Y and Y is a subset of X

Construction of sets-1

- Power set

  Powerset(X) = {Y | Y is a subset of X}

  - Notice that sizeof(Powerset(X)) = 2 raised to the power sizeof(X)

- Union

  union(X,Y) = {a | a in X or a in Y}

- Intersection

  intersection(X,Y) = {a | a in X and a in Y}

Construction of sets-2

- The elements of a set have no implied order.
  - That is, {a, b} = {b, a}
- We need to be able to specify the order of elements, so we define an ordered pair.

  (a,b) = { {a}, {a,b} }
  - Note: (b,a) = { {b}, {a,b}}
- We define a triple by

  (a,b,c) = (a,(b,c))
- and extend the concept to n-tuples

  (x1,x2,x3,…,xn) = (x1,(x2,(x3,(…(xn-1,xn)…))))

Construction of sets-3

- Product

  X x Y = {(a,b) | a in X and b in Y}

  X x Y x Z = {(a,b,c) | a in X, b in Y, c in Z}

- Disjoint Union

  disjointunion(X,Y) = union({0}xX,{1}xY)

- Set Difference

  X \ Y = { x | x in X and x not in Y}

Relations and Functions

Binary Relations

- A binary relation R between sets X and Y is an element of powerset(X x Y)
- That is, R is a subset of X x Y.
- If (x,y) in R, we write xRy
- ex. = is a binary relation on the natural numbers

Partial and Total Functions

- A <u>partial function</u> f from X to Y is a relation for which
  - (x,y) in f and (x,y') in f => y = y'
  - That is, f(x) has a unique value
- A <u>(total) function</u> f from X to Y is a partial function which is defined for every x in X.
  - x in X => there is a y in Y such that (x,y) in f

Composition of Relations (and Functions)

- Let R be a relation between sets X and Y and S be a relation between sets Y and Z. Then,
  S o R = {(x,z) | for some y in Y,
  (x,y) in R and (y,z) in S}

Equivalence Realtions

- Let R be a relation on X. (That is, R is a subset of X x X.)
- R is <u>reflexive</u> if xRx for all x in X
- R is <u>symmetric</u> if xRy => yRx
- R is <u>transitive</u> if xRy and yRz => xRz
- R is an <u>equivalence relation</u> if R is reflexive, symmetric, and transitive.

Equivalence Classes

- Let R be an equivalence relation on the set X. Then the R-equivalence class of x is { y | y in X and xRy }
- The equivalence classes induced on a set X are mutually inclusive and pairwise joint. That is, x in X belongs to exactly one equivalence class.

Examples of Binary Relations

On the integers

- = is an equivalence relation
- <= is reflexive and transitive but not symmetric
- < is transitive but not reflexive or symmetric