Binary Image Processing

- Introduction
- Set theory review
- Morphological filtering
  - Erosion and dilation
  - Opening and closing
  - Hit-or-miss, boundary extraction, ...
- Skeleton via distance transform

Binary Images

- Images only consist of two colors (tones): white or black

Numerical example (image of a square block)

```
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 255 255 255 255 0 0 0
0 0 255 255 255 255 0 0 0
0 0 255 255 255 255 0 0 0
0 0 255 255 255 255 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
```
**Binary Image Examples**

| Phil |

**Why are binary images special?**

- Since pixels are either white or black, the locations of white(black) pixels carry ALL information of binary images

Example

<table>
<thead>
<tr>
<th>0 0 0 0</th>
<th>0 0 255 255</th>
<th>0 0 255 255</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(m,n)</td>
<td>L={(3,3),(3,4),(4,3),(4,4)}</td>
<td>location of white pixels</td>
</tr>
</tbody>
</table>

It is often more convenient to consider the set representation than the matrix representation for binary images
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Set Theory Review

Think of sets $A$ and $B$ as the collections of spatial coordinates.
Translation Operator

\[(A)_z = \{w \mid w = a + z, a \in A\}\]

Example

Reflection Operator

\[\hat{B} = \{w \mid w = -b, b \in B\}\]

Example
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Structuring Element B

Definition: a set of local neighborhood with specified origin

Examples

\[
\begin{align*}
  B_1 & : & \begin{array}{c}
         \circ \circ \circ \\
         \circ \circ \circ \\
         \circ \circ \circ \\
       \end{array} \\
  B_2 & : & \begin{array}{c}
         \circ \circ \circ \\
         \circ \circ \circ \\
         \circ \circ \circ \\
       \end{array}
\end{align*}
\]

Note: different structuring element leads to different filtering result
**Erosion**

**Definition**

\[ Y = X \ominus B = \{ x : B_x \subset X \} \]

**Example**

![Erosion Example Diagram](image)

**Dilation**

**Definition**

\[ Y = X \oplus B = \{ z | (\hat{B})_z \cap X \neq \emptyset \} \]

or

\[ Y = X \oplus B = \bigcup_{b \in B} X_b = \bigcup_{x \in X} B_x = B \oplus X \]

**Example**

![Dilation Example Diagram](image)
**Theorem**

\[(X \ominus B)^c = X^c \oplus \mathring{B}\]

**Proof:**

\[(X \ominus B)^c = \{z \mid B \subseteq A\}^c\]

\[= \{z \mid B \cap A^c = \emptyset\}^c\]

\[= \{z \mid B \cap A^c \neq \emptyset\}\]

\[= X^c \oplus \mathring{B}\]
**Opening Operator**

**Definition**

\[ X \circ B = (X \ominus B) \oplus B \]

**Example**

- \( X \)
- \( X \circ B \)
- Mask \( B \)

**Geometric Interpretation of Opening Operator**

- Translates of \( B \) in \( A \)
- \( A \circ B = \cup \{(B)_t : (B)_t \subseteq A\} \)
**Closing Operator**

**Definition**

\[ X \bullet B = (X \oplus B) \ominus B \]

**Example**

- **X**
  - ![X Image](image1)
- **X \bullet B**
  - ![X*B Image](image2)

**Geometric Interpretation of Closing Operator**

- ![Geometric Image](image3)
Properties of Opening and Closing Operators*

**Opening**
- $X \circ B \subseteq X$
- $X \subseteq Y \Rightarrow X \circ B \subseteq Y \circ B$
- $(X \circ B) \circ B = X \circ B$

**Closing**
- $X \subseteq X \bullet B$
- $X \subseteq Y \Rightarrow X \bullet B \subseteq Y \bullet B$
- $(X \bullet B) \bullet B = X \bullet B$

---

**Hit-or-Miss Operator**

**Definition**

$X \bullet B = (X \ominus B_1) \cap (X^c \ominus B_2)$

**Structuring element example**

<table>
<thead>
<tr>
<th>Mask B_1</th>
<th>Mask B_2</th>
<th>Mask B</th>
<th>MATLAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 -1 -1</td>
<td>1 1 -1</td>
<td>0 1 0</td>
<td>bwhitmiss</td>
</tr>
</tbody>
</table>

(MATLAB function: bwhitmiss)
**Example**

\[ X \quad X' \quad X \ominus B_1 \quad X' \ominus B_2 \]

- \( X \)
- \( X' \)
- \( X \ominus B_1 \)
- \( X' \ominus B_2 \)

- **Boundary Extraction Operator**

Definition

\[ \partial X = X - (X \ominus B) \]

Example

\[
\begin{array}{c}
\uparrow \\
X \\
\downarrow \\
X \ominus B \\
\downarrow \\
X - (X \ominus B) \\
\end{array}
\]

MATLAB function: `bwmorph(…,'remove')`
**Image Example**

**Region Filling Operator***

Idea: recursively expand the region around $P$ but stop the expansion at the boundary of $X$

Iterations:

- $Y_0 = P$
- $Y_k = (Y_{k-1} \oplus B) \cap X^c$, $k=1,2,3\ldots$

Terminate when $Y_k = Y_{k-1}$, output $Y_k \cup X$
**Image Example**

![Image Example Diagram]

**Additional Example**

![Additional Example Diagram]
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Medial Axis (Skeleton)

- Definition
  Suppose that a fire line propagates with constant speed from the contour of a connected object towards its inside, then all those points lying in positions where at least two wave fronts of the fire line meet during the propagation will constitute a form of a skeleton

- Examples
Skeleton Algorithm

- Distance transform: find the distance from the nearest boundary for each point
  - initialization \( x_0(m,n) = x(m,n) \)
  - iteration \( x_k(m,n) = x_k(m,n) + \min \{ x_{k-1}(i,j) : d(m,n;i,j) \leq 1 \} \)

- Skeleton is the set of points whose distance from the nearest boundary is locally maximum
  \( \{(m,n) : x_k(m,n) \geq x_k(i,j), d(m,n;i,j) \leq 1 \} \)

Numerical Example

\[
\begin{array}{ccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 2 & 2 & 1 & 1 & 1 \\
1 & 2 & 2 & 2 & 1 & 2 & 3 \\
1 & 2 & 2 & 2 & 1 & 2 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

\[
x_0(m,n) \quad x_1(m,n) \quad x_3(m,n) \quad \text{skeleton}
\]

\[
x_0(m,n) \quad x_1(m,n) \quad x_3(m,n) \quad \text{local maximum}
\]
Image Example

original  skeleton

MATLAB code: `y=bwmorph(x,'skel',inf);`

Image Example (Con’t)

Binaried fingerprint image  Skeleton image